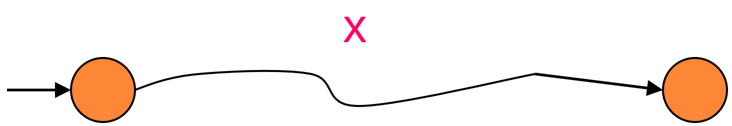
Conversion From NFA to DFA

DFA

- For every string x, there is a unique path from initial state and associated with x.



x is accepted if and only if this path ends at a final state.

NFA

- For any string x, there may exist none or more than one path from initial state and associated with x.

NFA → DFA

- Consider an NFA M=(Q, Σ, δ, s, F).
- For x in Σ*, define

  [x] = {q in Q | there exists a path s $\xrightarrow{x}$ q}
- Define DFA M'=(Q', Σ, δ', s', F'}:

  Q' = { [x] | x in Σ* },

  δ ([x], a) = [xa] for x in Σ* and a in Σ,

  s' = [ε],

  F' = { [x] | x in L(M) }

Construction of M'

Special Case: M has no ε-move.

- [ε] = {s}
- Suppose [x] is known. How to get [xa] for a in Σ?

From [x] to [xa]

- $[xa] = \{\, p \mid$ there exists a path $q \xrightarrow[\text{path}]{a} p$ for some q in $[x]\,\}$

  $= \{\, p \mid$ there exists q in $[x]$, $q \xrightarrow[\text{edge}]{a} p\,\}$

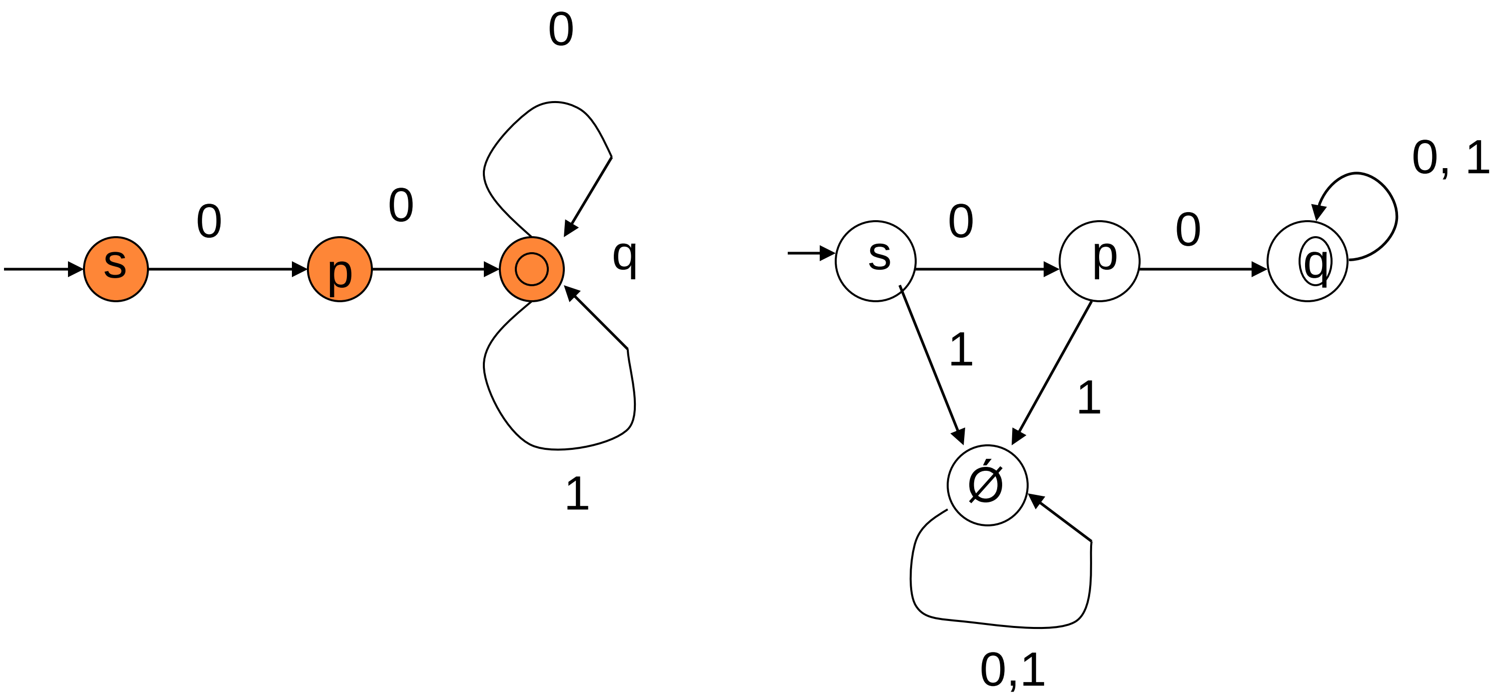
  $= \bigcup_{q \text{ in } [x]} \delta(q,a)$

Construction of M'

- $F' = \{[x] \mid x \text{ in } L(M)\}$
  $= \{[x] \mid [x] \cap F \neq \emptyset\}$

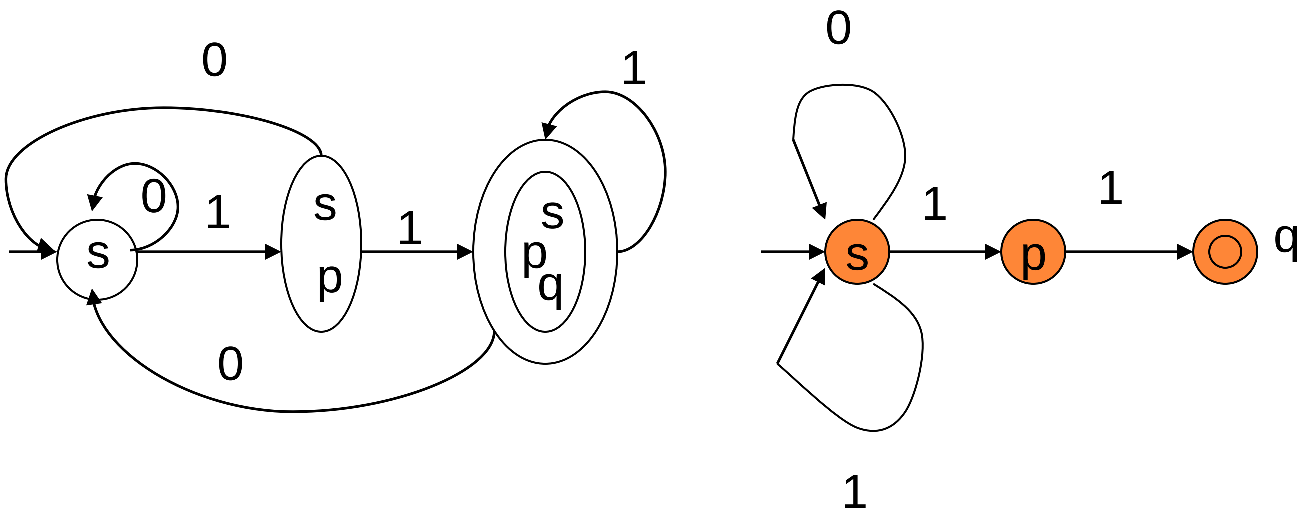
EXAMPLE 1

- Construct DFA to accept 00(0+1)*



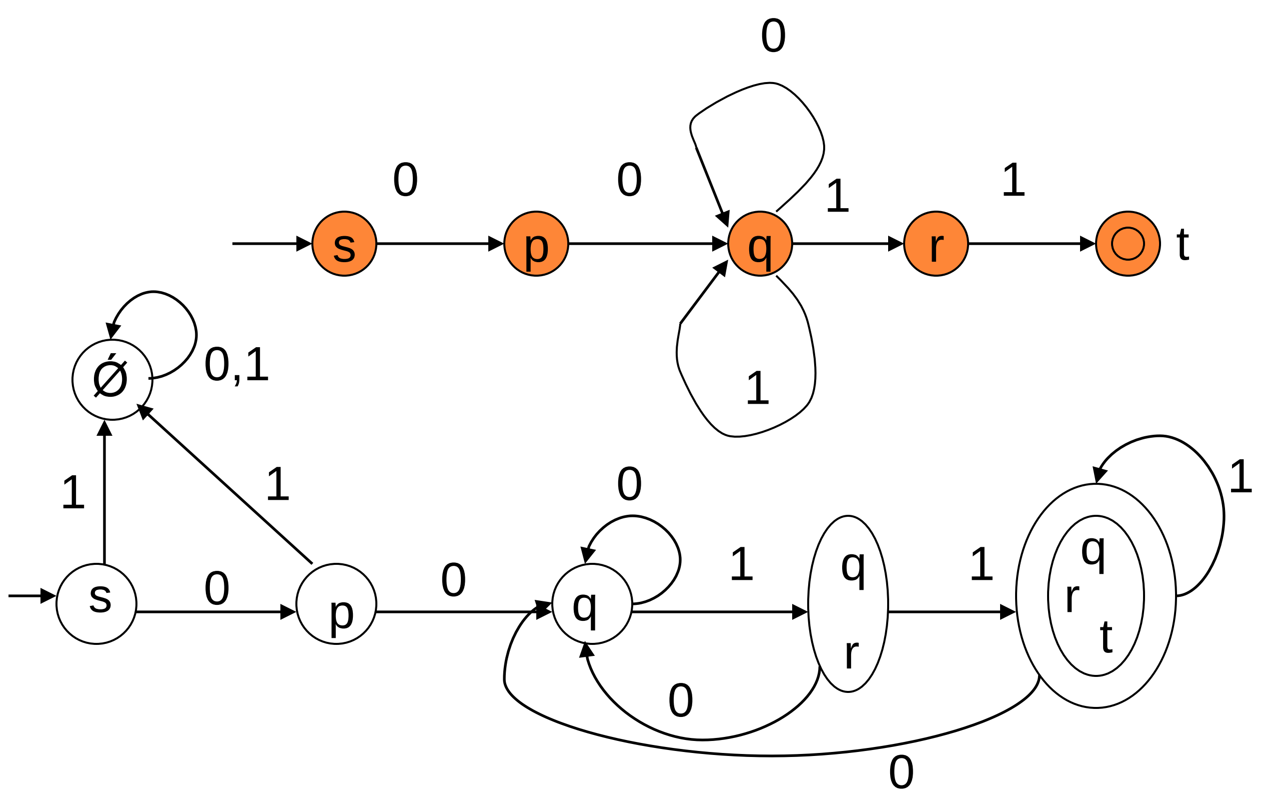
EXAMPLE 2

- Design DFA to accept (0+1)*11



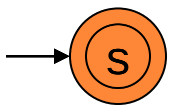
EXAMPLE 3

- Design DFA to accept 00(0+1)*11

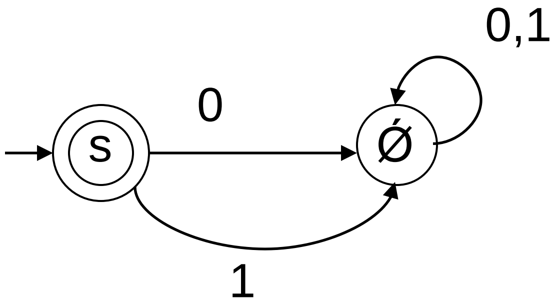


Example 4

- Construct DFA M for L(M)=ε.



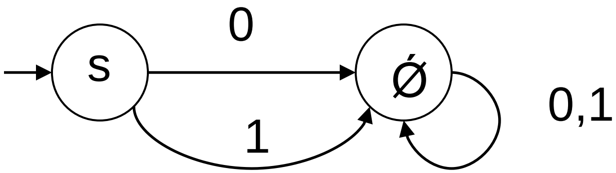
Is this a DFA?

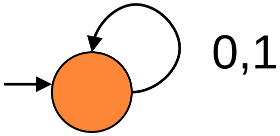


EXAMPLE 5

- Construct DFA M for L(M)=Ø.

Is it a DFA?





CONSTRUCTION OF M’

- For q in Q, define

ε-closure(q) = {p | there exists a path $q \xrightarrow[\text{path}]{\varepsilon} p$}

- [ε] = {q | there is a path $s \xrightarrow[\text{path}]{\varepsilon} q$}

  = ε-closure(s)

- Suppose [x] is known. How to get [xa] for a in Σ?

From [x] to [xa]

- [xa] = { p | there exists a path $q \xrightarrow[\text{path}]{a} p$ for some q in [x] }

  = { p | there exists q in [x], $q \xrightarrow[\text{edge}]{a} r \xrightarrow[\text{path}]{\varepsilon} p$ }

  = { p | for some q in [x] and r in δ(q,a), p in ε-closure(r) }

  $= \bigcup_{q \text{ in } [x]} \bigcup_{r \text{ in } \delta(q,a)} \varepsilon\text{-closure}(r)$

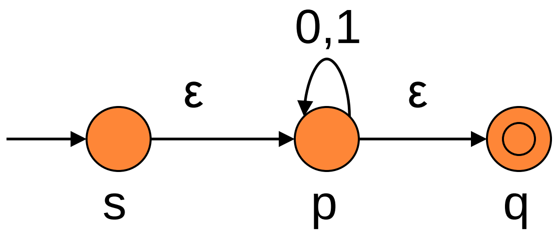
Construction of M'

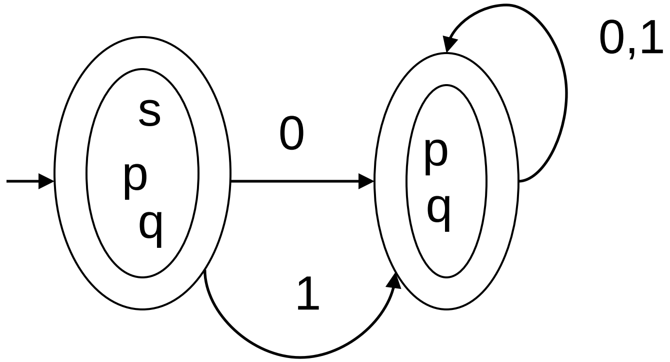
- F' = {[x] | x in L(M)}

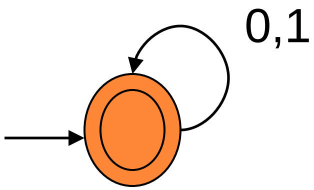
  = {[x] | [x] ∩ F ≠ Ǿ }

EXAMPLE 6

- Construct DFA M for L(M)=(0+1)*.

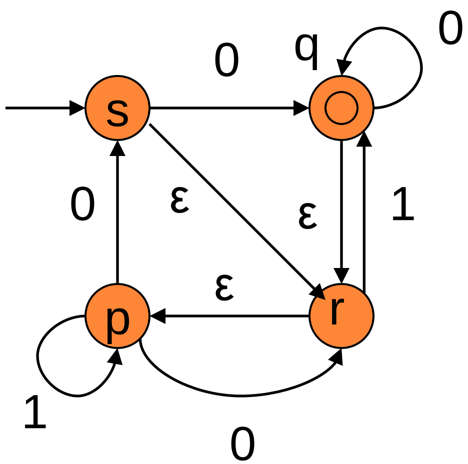


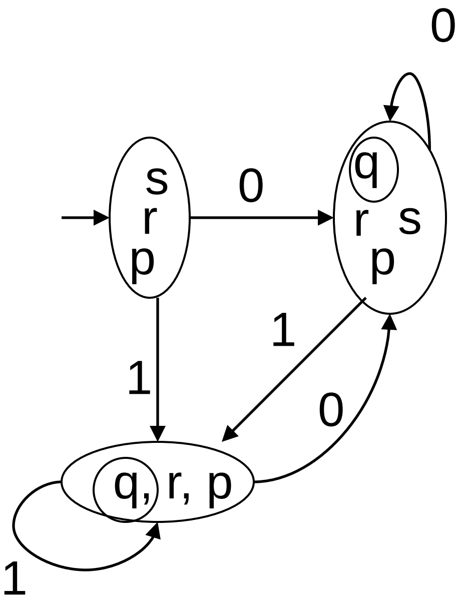


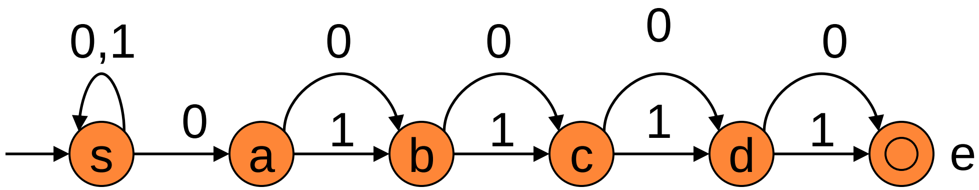


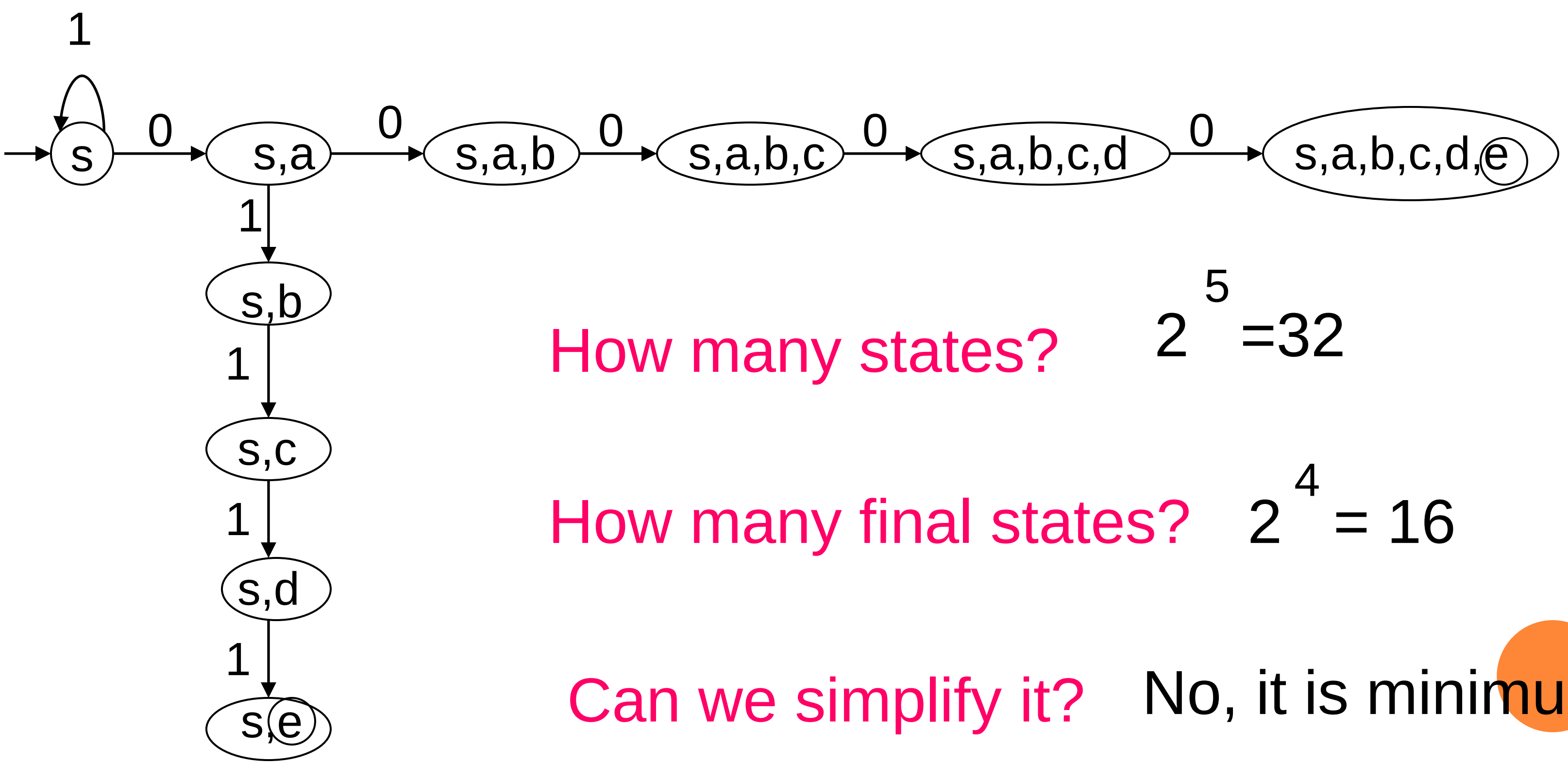
EXAMPLE 7

- Convert the following NFA to DFA.









How many states? $2^5 = 32$

How many final states? $2^4 = 16$

Can we simplify it? No, it is minimum!